NUMERICAL ANALYSIS IN BK EVOLUTION WITH IMPACT PARAMETER

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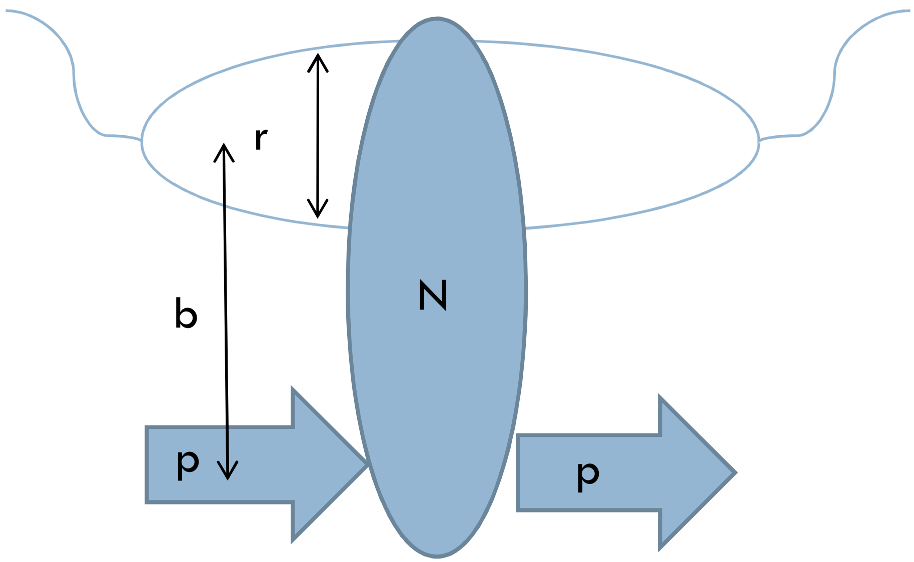
Dipole Model



Photon splits into a color dipole of size r which interacts at impact parameter b with the target (nucleon)

Color dipole interacts with partons of the target through gluon exchange

N(r,b,Y) is the scattering amplitude of the dipole interaction

[A.H.Mueller, Nucl. Phys B415 373 (1994)]

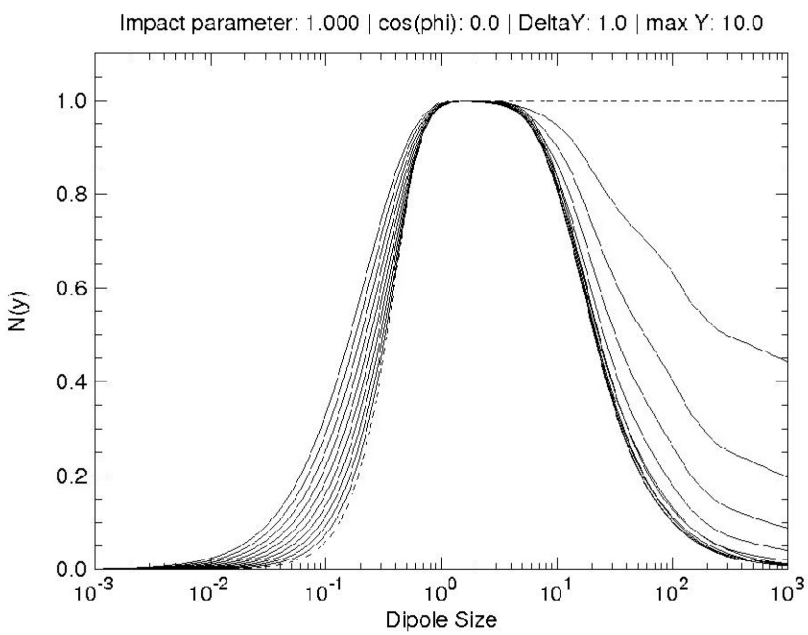
- This analysis is done in the context of the dipole model of small x scattering. In this regime the evolution of the amplitude can be represented as a dipole cascade.

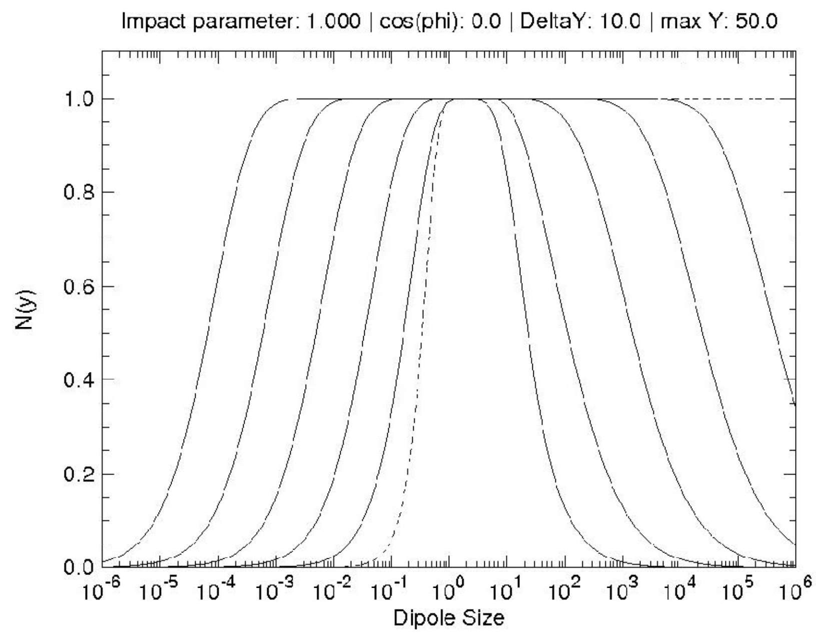
The BK equation

$$\frac{\partial N_{01}}{\partial Y} = \alpha_s \int d^2\boldsymbol{x_2} K[N_{02} + N_{12} - N_{01} - N_{02}N_{12}]$$

- Enforces unitarity in the amplitude $N_{ij} = N(x_{ij}, b_{ij}, \vartheta_{ij}, Y)$
- Parent dipole $\boldsymbol{x_{01}} = \boldsymbol{x_0} - \boldsymbol{x_1}$ splits into two dipoles of $\boldsymbol{x_{02}}$ and $\boldsymbol{x_{12}}$
- Splitting is determined by the kernel $K = K(x_{01}, x_{02}, x_{12})$
- Impact parameter $\boldsymbol{b_{ij}} = \frac{1}{2}(\boldsymbol{x}_i + \boldsymbol{x}_j)$ only dependence is in the amplitude
- Angle ϑ_{ij} is the angle between $\boldsymbol{x_{ij}}$ and $\boldsymbol{b_{ij}}$
- Usually the amplitude is assumed uniform in impact parameter, here we take the full dependences of the amplitude on impact parameter into account

Features of BK with impact parameter

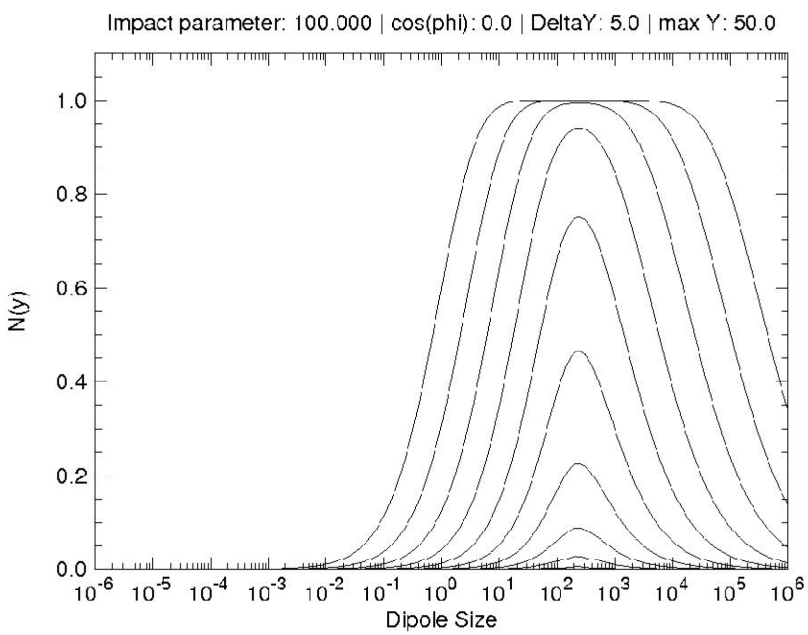


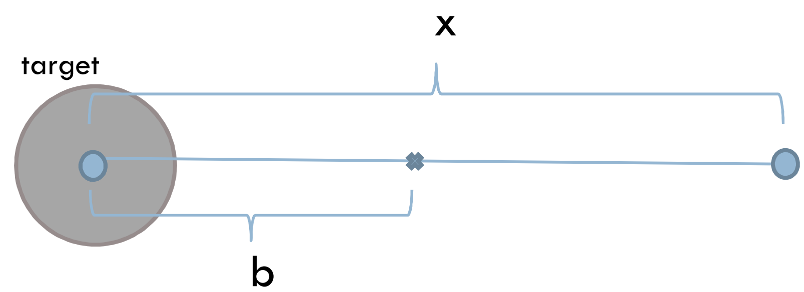


- Leading order kernel used
- Coupling fixed at $\frac{N_c \alpha_s}{\pi} = 0.1$

$$K = \frac{dz}{z} \frac{N_c \alpha_s}{2\pi^2} \frac{x_{01}^2}{x_{02}^2 x_{12}^2}$$

Large contributions at x = 2b



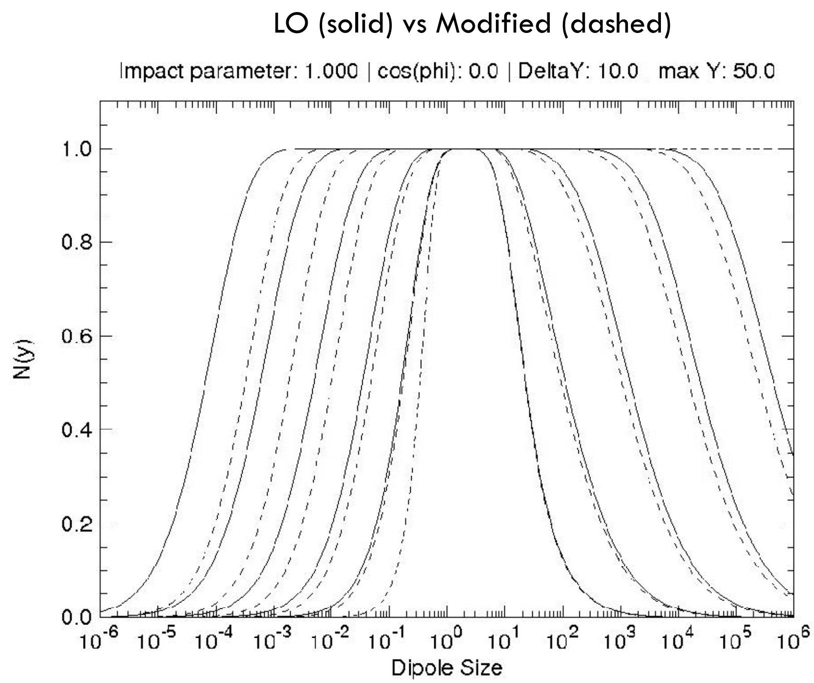


Nontrivial angular dependence.

Peak of the amplitude occurs when $x = 2b$ and $\boldsymbol{x} \parallel 2\boldsymbol{b}$

- This behavior can be extracted from the representation in terms conformal eigenfunctions

Towards higher order



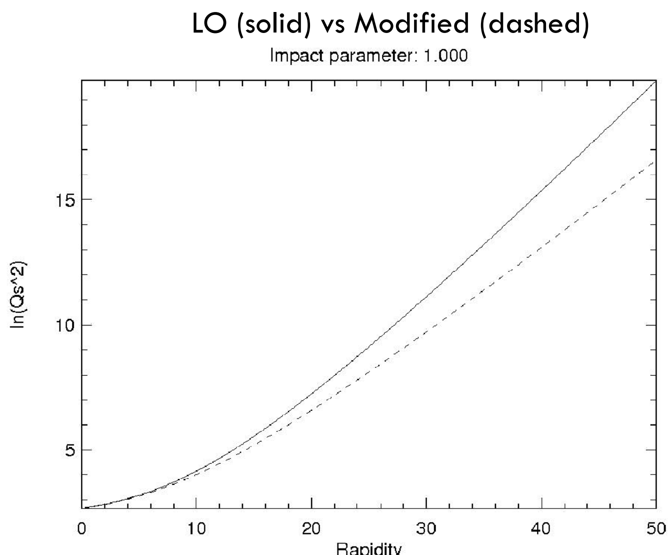
$$K = \frac{dz}{z}\frac{N_c\alpha_s}{2\pi^2}\frac{z}{x_{01}^2}\left[K_1^2\left(\frac{x_{02}}{x_{01}}\sqrt{z}\right)+K_1^2\left(\frac{x_{12}}{x_{01}}\sqrt{z}\right)-\frac{2x_{02}\cdot x_{12}}{x_{02}x_{12}}K_1\left(\frac{x_{02}}{x_{01}}\sqrt{z}\right)K_1\left(\frac{x_{12}}{x_{01}}\sqrt{z}\right)\right]$$

[L. Motyka and A. M. Stasto, Phys. Rev. D79, 085016 (2009)]

This kernel reduces to the LO kernel at large rapidies or when $x_{01} >> x_{02}, x_{12}$

- Kinematical cut owing to a modification in the energy denominator
- The modified kernel slows the evolution by approximately 30%
- The modified kernel has almost no affect when the impact parameter dependence is neglected due to the saturation of all large dipole sizes.

Saturation Scale



$$\left\langle N\left(r = 1/Q_s(b,Y), b, \theta, Y\right)\right\rangle = 0.5$$

Saturation scale was found to have the same impact parameter dependence at large b which leads us to a factorized form

$$Q_s^2(b,Y) = Q_0^2 e^{\bar{\alpha}_s \lambda_s Y} S(b) \qquad S(b) \sim \frac{1}{b^4}$$

	LO	Modified
λ_s	4.4	3.6 $\bar{\alpha}_s = 0.1$ (2.5 $\bar{\alpha}_s = 0.2$)

- Saturation is when the parton density becomes large and recombination effects become important
  - Defined here as the amplitude becomes large and the nonlinear term becomes important.
- Numbers are consistent with analytical estimates

[S. Munier and R. B. Peschanski, Phys. Rev. D69, 034008 (2004)]

[A. H. Mueller and D. N. Triantafyllopoulos, Nucl. Phys. B640, 331]

Running coupling

- Several different prescriptions for running coupling
- Balitsky

$$K=\frac{dz}{z}\frac{N_c\alpha_s(x_{01}^2)}{2\pi^2}\left[\frac{x_{01}^2}{x_{02}^2x_{12}^2}+\frac{1}{x_{02}^2}\left(\frac{\alpha_s(x_{02}^2)}{\alpha_s(x_{12}^2)}-1\right)+\frac{1}{x_{12}^2}\left(\frac{\alpha_s(x_{12}^2)}{\alpha_s(x_{02}^2)}-1\right)\right]$$

[I. Balitsky, Phys. Rev. D75, 014001 (2007)]

- Kovchegov -Weigert

$$K=\frac{dz}{z}\frac{N_c}{2\pi^2}\left[\frac{1}{x_{20}^2}\alpha_s\left(\frac{4e^{-5/3-2\gamma}}{x_{20}^2}\right)+\frac{1}{x_{12}^2}\alpha_s\left(\frac{4e^{-5/3-2\gamma}}{x_{12}^2}\right)-2\frac{\boldsymbol{x}_{12}\cdot\boldsymbol{x}_{20}}{x_{20}^2x_{12}^2}\frac{\alpha_s\left(\frac{4e^{-5/3-2\gamma}}{x_{20}^2}\right)\alpha_s\left(\frac{4e^{-5/3-2\gamma}}{x_{20}^2}\right)}{\alpha_s\left(\frac{4e^{-5/3-2\gamma}}{R^2(x_0,x_1;x_2)}\right)}\right]$$

[Y. V. Kovchegov and H. Weigert, Nucl. Phys. A784, 188 (2007)]

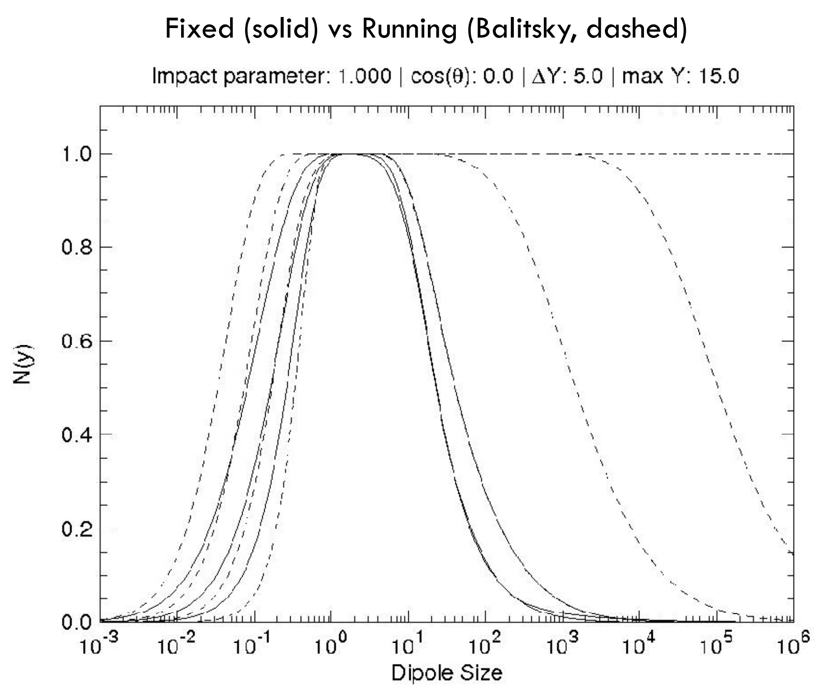
- Parent Dipole

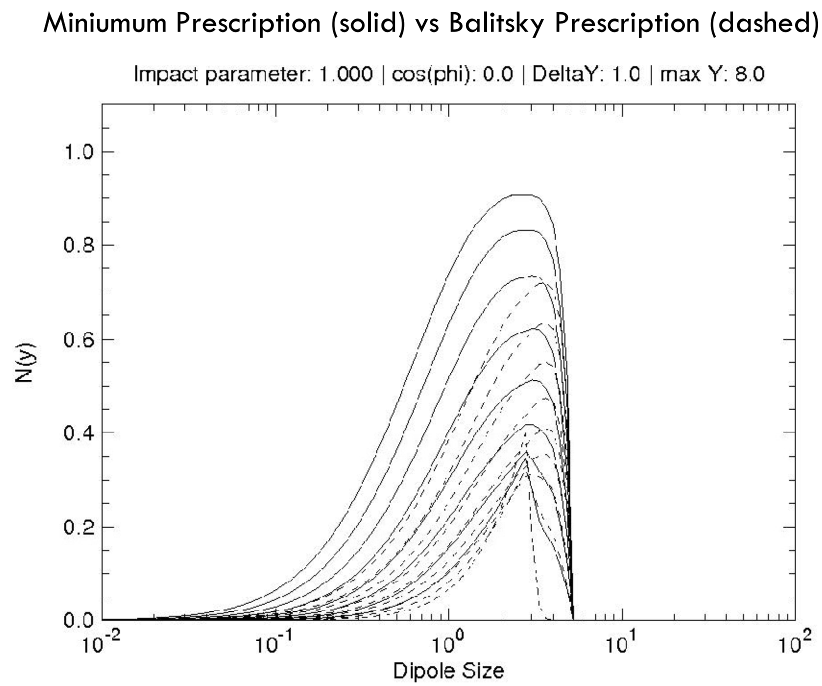
$$K=\frac{dz}{z}\frac{N_c\alpha_s\left(x_{01}^2\right)}{2\pi^2}\frac{x_{01}^2}{x_{02}^2x_{12}^2}$$

- Minimum Dipole

$$K=\frac{dz}{z}\frac{N_c\alpha_s\left(\min\left(x_{01}^2,x_{12}^2,x_{02}^2\right)\right)}{2\pi^2}\frac{x_{01}^2}{x_{02}^2x_{12}^2}$$

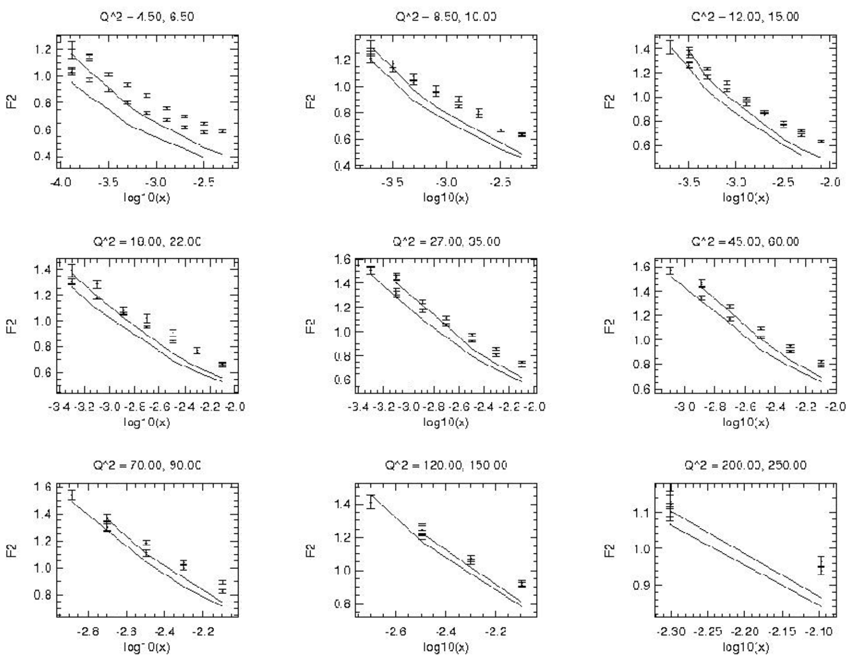
Results with running coupling





- IR regularization of the kernel is important due to large dipole evolution
- Balitsky's running coupling is well slower than the minimum dipole prescription

F_2



Fixed coupling kernels evolve too fast unless coupling is artificially low

Minimum dipole prescription is also too fast

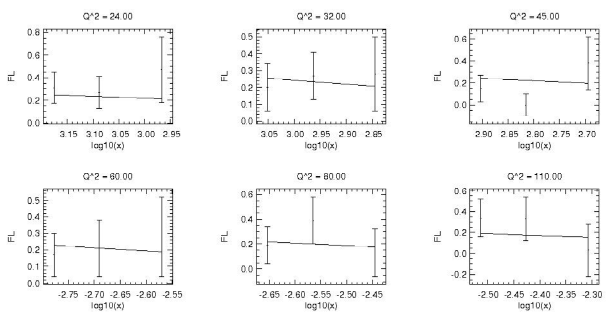
- The prescription by Balitsky for running coupling has unusual properties
  - Slower than expected from the momentum space analysis
  - Extremely sensitive to the form of regularization of $\alpha_s(x^2)$
  - Closeness to the data should perhaps be regarded as accidental at this time

F_2 & F_L

F_2

- In general the slope is too steep to fit the data
- Data is underestimated due to lack of contribution from large dipole sizes
- Need a separate contribution due to these large, non-perturbative dipoles

F_L



- F_L data is not very discriminatory due to large error bars

Conclusions

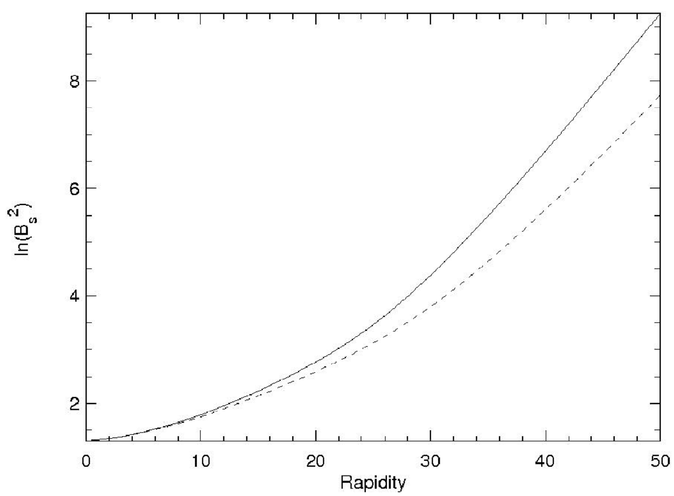
- Solving the BK equation with impact parameter is crucial – many features are left out otherwise!
  - N → 0 for large dipole sizes
  - Amplitude enhanced at x = 2b with peaks at $\cos(\theta) = +1, -1$
  - Power tails in impact parameter
  - Second wavefront develops evolving to larger dipole size
- Running coupling prescriptions slow the evolution more than expected, bringing us surprisingly close to the data, however there is a large sensitivity to regularization as well as unexpected behavior.

Thank You

Special Thanks to : My advisor Anna Stasto as well as Henry Kowalski for discussions and use of his code and Emil Avsar for interesting discussions.

Backup Slides

Diffusion in impact parameter



$$\langle N(r, B_s = b, \theta, Y)\rangle = 0.5$$

Growth of the black disk corresponds to growth of the cross section

$$B_s^2(r,Y) = B_{s0}^2 e^{\bar{\alpha}_s \lambda_{sB} Y} F(r) \qquad \sigma \approx e^{2\lambda_{sB} Y}$$

	LO	Modified
λ_{sB}	2.6	2.2 $\bar{\alpha}_s = 0.1$ (2.0 $\bar{\alpha}_s = 0.2$)

- Increasing energy causes the dense region of the dipole cascade to expand in impact parameter space
- Size of the dense or ‘black’ region characterized by a radius of this black disk
- Fast increase in is partially due to the lack of scale in the solution currently

Adding mass parameter

- Full cut with theta function

$$K = \frac{dz}{z}\frac{N_c \alpha_s}{2\pi^2}\frac{x_{01}^2}{x_{02}^2 x_{12}^2}\theta\left(\tfrac{1}{m^2} - x_{02}^2\right)\theta\left(\tfrac{1}{m^2} - x_{12}^2\right)$$

- Splitting the theta function

$$K = \frac{dz}{z}\frac{N_c \alpha_s}{2\pi^2}\left[\frac{1}{x_{02}^2}\theta\left(\tfrac{1}{m^2} - x_{02}^2\right) + \frac{1}{x_{12}^2}\theta\left(\tfrac{1}{m^2} - x_{12}^2\right) - 2\frac{x_{02}\cdot x_{12}}{x_{02}^2 x_{12}^2}\theta\left(\tfrac{1}{m^2} - x_{12}^2\right)\theta\left(\tfrac{1}{m^2} - x_{02}^2\right)\right]$$

- Bessel function cut

$$K = \frac{dz}{z}\frac{N_c \alpha_s m^2}{2\pi^2}\left[K_1^2(mx_{02}) + K_1^2(mx_{12}) - 2K_1(mx_{02})K_1(mx_{12})\frac{x_{02}\cdot x_{12}}{x_{02}x_{12}}\right]$$

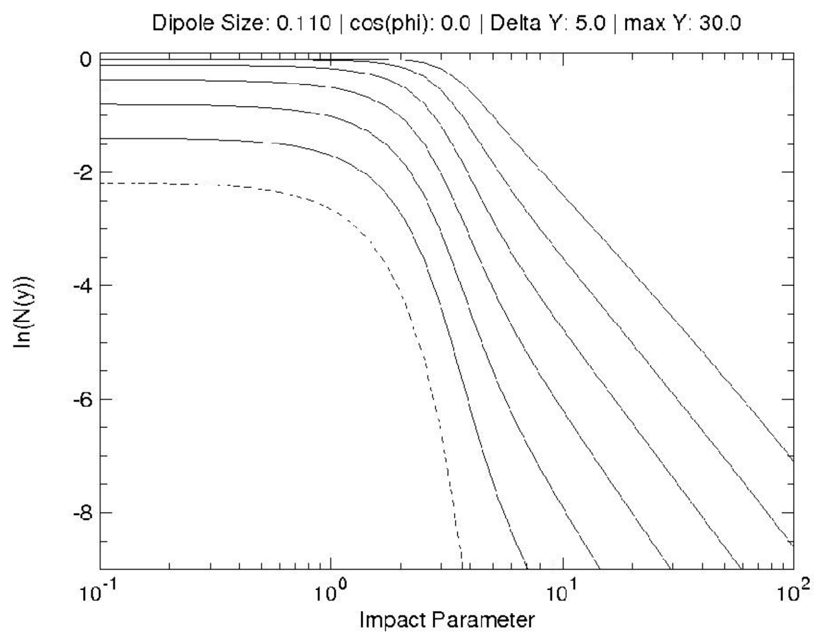
- Running coupling with theta function

$$K = \frac{dz}{z}\frac{N_c \alpha_s(x_{01}^2)}{2\pi^2}\left[\frac{x_{01}^2}{x_{02}^2 x_{12}^2} + \frac{1}{x_{02}^2}\left(\frac{\alpha_s(x_{02}^2)}{\alpha_s(x_{12}^2)} - 1\right) + \frac{1}{x_{12}^2}\left(\frac{\alpha_s(x_{12}^2)}{\alpha_s(x_{02}^2)} - 1\right)\right]\theta\left(\tfrac{1}{m^2} - x_{12}^2\right)\theta\left(\tfrac{1}{m^2} - x_{02}^2\right)$$

- Modified kernel with theta function

$$K = \frac{dz}{z}\frac{N_c \alpha_s}{2\pi^2}\frac{z}{x_{01}^2}\left[K_1^2\left(\frac{x_{02}}{x_{01}}\sqrt{z}\right) + K_1^2\left(\frac{x_{12}}{x_{01}}\sqrt{z}\right) - \frac{2x_{02}\cdot x_{12}}{x_{02}x_{12}}K_1\left(\frac{x_{02}}{x_{01}}\sqrt{z}\right)K_1\left(\frac{x_{12}}{x_{01}}\sqrt{z}\right)\right]\theta\left(\tfrac{1}{m^2} - x_{12}^2\right)\theta\left(\tfrac{1}{m^2} - x_{02}^2\right)$$

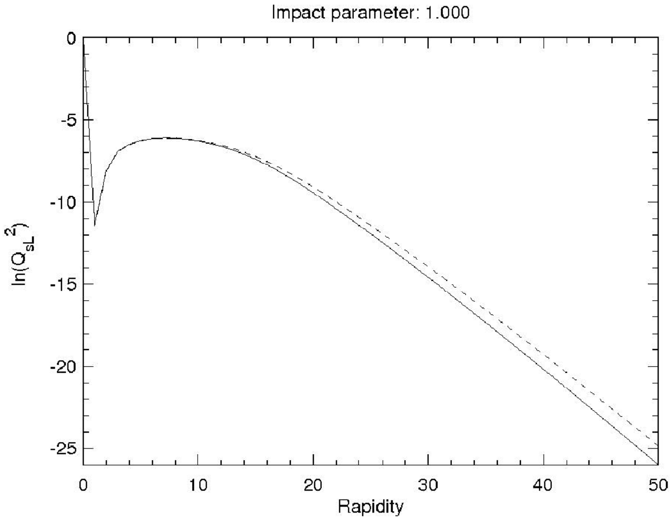
Impact parameter tails



- Power-like tails are generated during the evolution
- Initial impact parameter dependence $N = 1 - e^{-x^2 e^{-b^2}}$ is quickly forgotten
- There is a clear 'ankle' where dependences of the amplitude on impact parameter become power-like

A Second Saturation Scale

LO (solid) vs Modified (dashed)



$$\left\langle N\left(r = 1/_{Q_{sL}(b,Y)}, b, \theta, Y\right)\right\rangle = 0.5$$

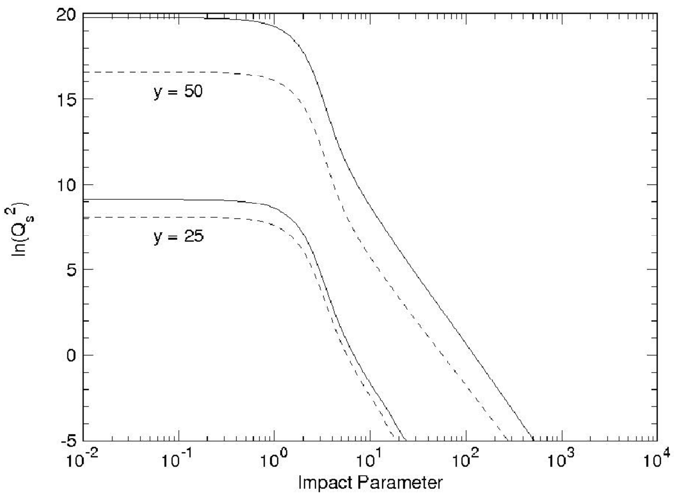
Equation has two solutions now! Same Parameterization

$$Q_{sL}^2(b,Y) = Q_{0L}^2 e^{-\bar{\alpha}_s \lambda_{sL} Y} S_L(b)$$

	LO	Modified
λ_{sL}	6.0	5.8 $\bar{\alpha}_s = 0.1$ (5.2 $\bar{\alpha}_s = 0.2$)

- Larger dipole sizes have slightly different saturation scale exponents
  - More thinking to be done on this result…

Saturation Scale – B dependence



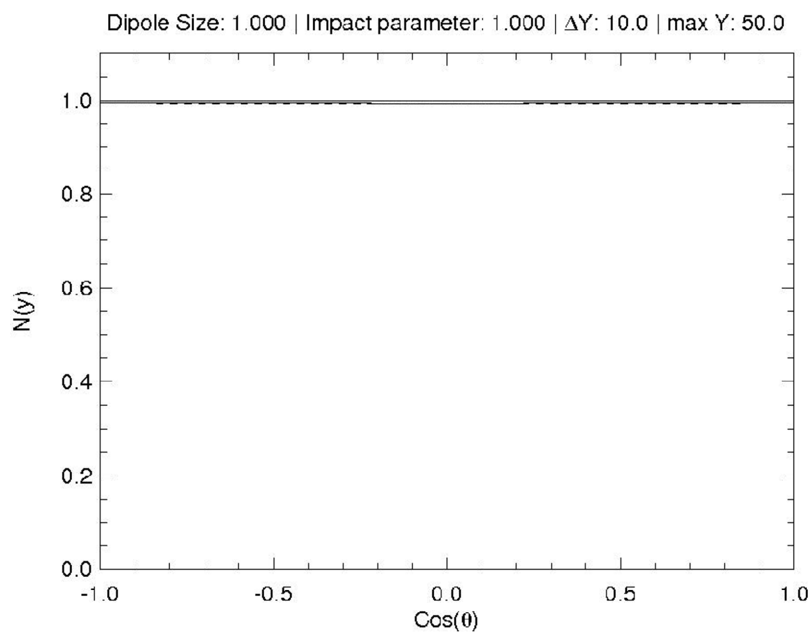
$$\left\langle N\left(r = 1/Q_s(b,Y), b, \theta, Y\right)\right\rangle = 0.5$$

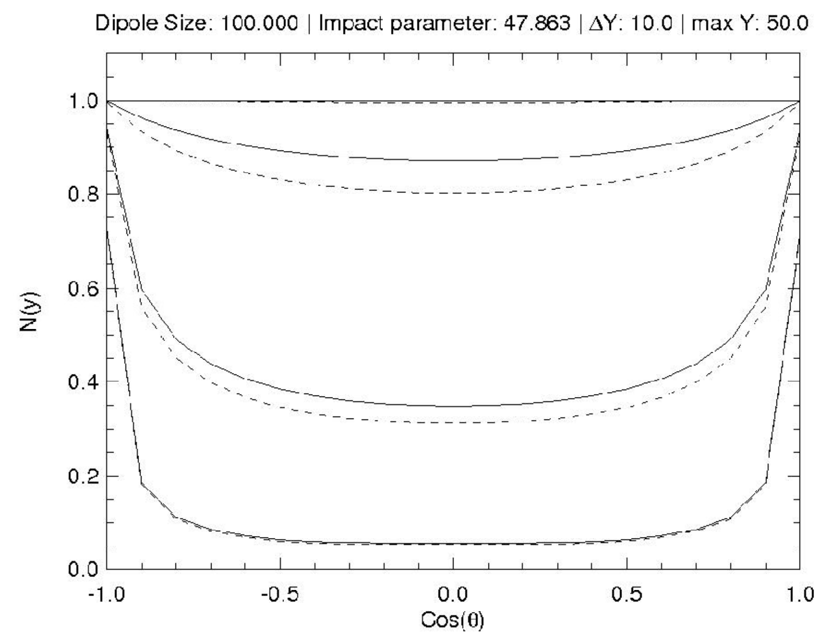
Saturation scale was found to have the same impact parameter dependence at large b which leads us to a factorized form

$$Q_s^2(b,Y) = Q_0^2 e^{\overline{\alpha}_s \lambda_s Y} S(b)$$

- Large impact parameters yield similar slopes with similar dependences

Angular Dependence

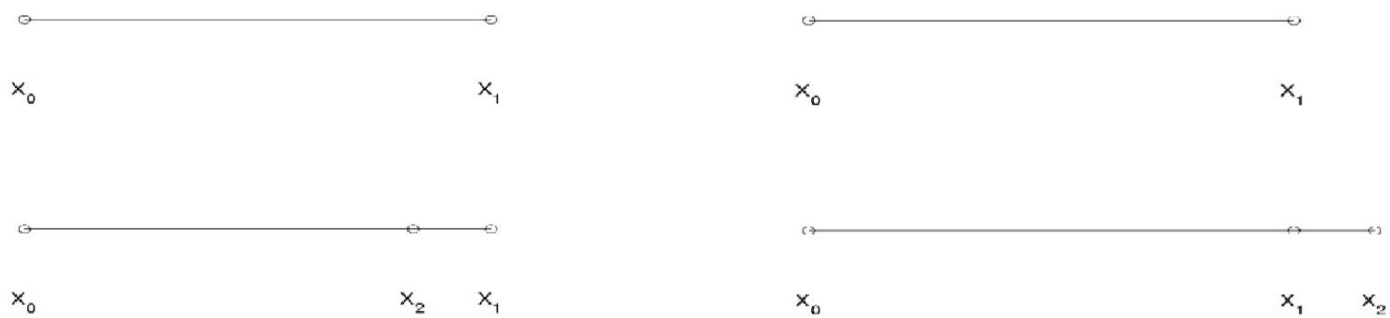




- Angular dependence only comes in when x = 2b
- Enhancements when $\cos(\theta) = +1, -1$

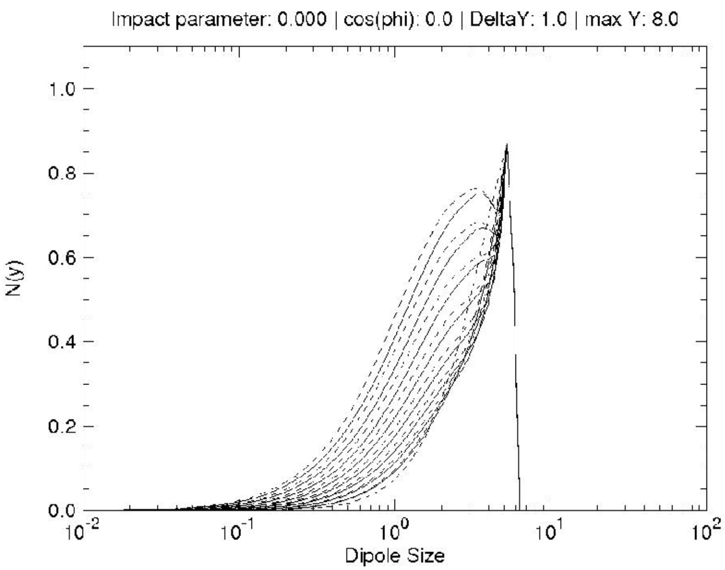
Unusual slowness of the coupling

- Naïve analysis leads us to believe the equivalence of the minimum dipole size coupling and Balitsky's
- Numerical analysis reveals this not to be true



When one daughter dipole is small there are regions where one prescription dominates when $\cos(\theta) = +1$ [left] the minimum dipole size method dominates while when $\cos(\theta) = -1$ [right] the Balitsky prescription for running coupling dominates, however these regions are not equal in BK.

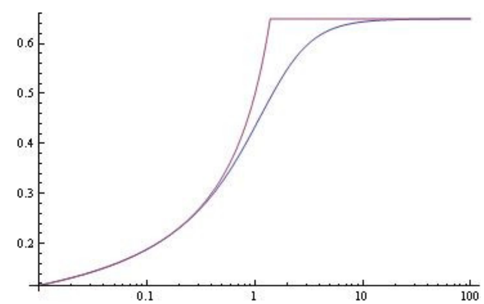
Surprising behaviors of Balitsky's kernel



$$\alpha_s(x^2) = \frac{1}{b \ln\left(\frac{1}{\Lambda^2}\left(\frac{1}{x^2} + \mu^2\right)\right)}$$

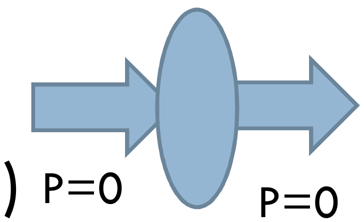
Using a μ factor to regularize the coupling or a sharp cutoff was found to change the amplitude by much more than expected (a factor of 2 or more in some cases), indicating a great sensitivity to the specific form the coupling takes.

Increasing the μ decreases the coupling but in the case of the Balitsky kernel this increases the amplitude

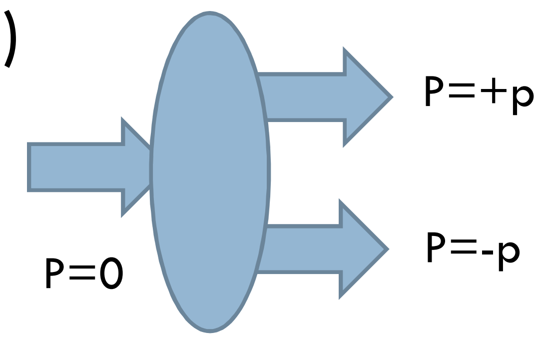


Impact Parameter is so importiant!

- Impact parameter corresponds to momentum transfer, neglecting impact parameter is equivalent to setting momentum transfer → 0
- With BFKL this is self consistent
  - Only linear terms (two pomeron vertex)

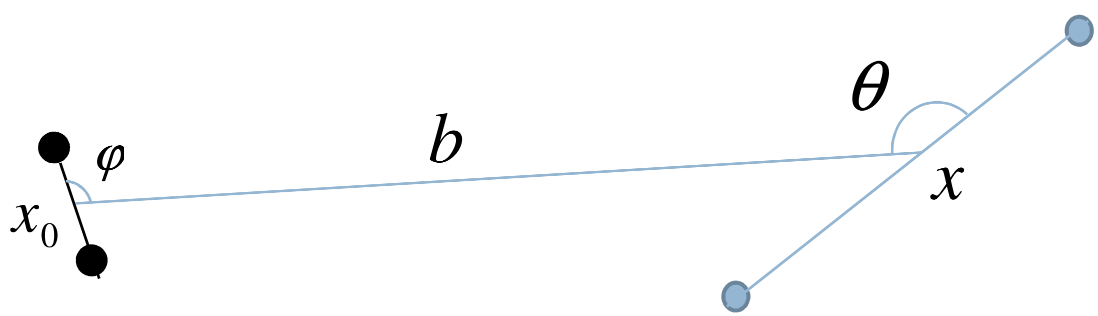


- This assumption with BK causes problems
  - Nonlinear term (three pomeron vertex)
  - Momentum transfer cannot stay zero without altering the interaction



Conformal Symmetry?

- LO Kernel is conformally invariant
- Expect evolution in small dipole and large dipole directions to be the same
- Additional angular dependence? Numerics say no dice



- Need higher order corrections?